Quantum Critical Point: A Comparison of Theory And Experiments

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QUEST 2003

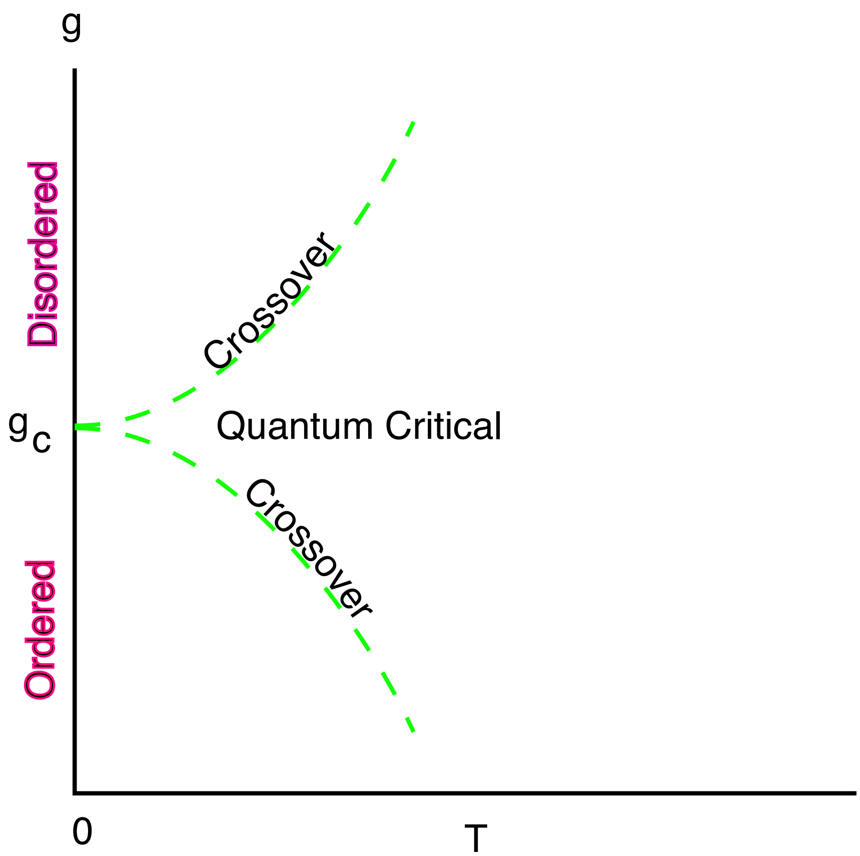
- QCP: Definition
- A simple example
- How is a QCP detected in a finite temperature measurement?
- Experiments — evidence of a ubiquitous metallic state
- Role of dissipation — change of the universality class

Definition

- Classical critical point—thermal fluctuations—scale invariance—divergent correlation length. Free energy is a non-analytic function at $T = T_c$.

- Quantum critical point—quantum fluctuations at $T = 0$—scale invariance—divergent correlation length. Ground state energy is a non-analytic function of tuning parameter $g = g_c$.

g may be charging energy in a Josephson junction array, magnetic field in a field tuned superconductor-insulator transition, or in a quantum Hall plateau transition, or doping (which destroys antiferromagnetism) in the parent high T_c superconductor, or disorder in a conductor in a metal-insulator transition.



Correlation length: $\xi \sim \frac{1}{|g-g_c|^{\nu}}$

Correlation time: $\xi_\tau \sim \xi^z$

ν: correlation length exponent

z: dynamical exponent

1D Ising model in a transverse field: a simple example

$$H = -h \sum_i \sigma_i^x - J \sum_i \sigma_i^z \sigma_{i+1}^z$$

When the transverse field $h \to \infty$, the ground state is disordered. Each site is independent of the other, and there is an excitation gap of $2h$ between the state $\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$ and $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$. When $J \to \infty$, the ground state is ordered. The ground state corresponds to all spins, $|\uparrow\rangle$, or all spins down, $|\downarrow\rangle$, reflecting broken symmetry. There is a $T = 0$ phase transition at a critical value of $g_c = (h/J)_c$, which is in the same universality class as the 2D Ising model solved by Onsager. The correlation length exponent $\nu = 1$. The dynamic exponent $z = 1$ (massless Dirac fermions at g_c).

There is more to it than meets the blinking eye.

●Classical statistical mechanics — dynamics and thermodynamics are independent of each other.

●Quantum mechanics — dynamics and thermodynamics are intimately tied to each other.

Classes of quantum phase transition, which do not have classical analogs.

1. Localization of an electron in a random potential (Anderson localization)—ground state energy is an analytic function of disorder—yet insulating and metallic phases are distinct states of matter. Also integer quantum Hall plateau transitions.

2. Quantum phase transition involving Berry phases.

3. Topological phase transitions on a non-trivial manifold.

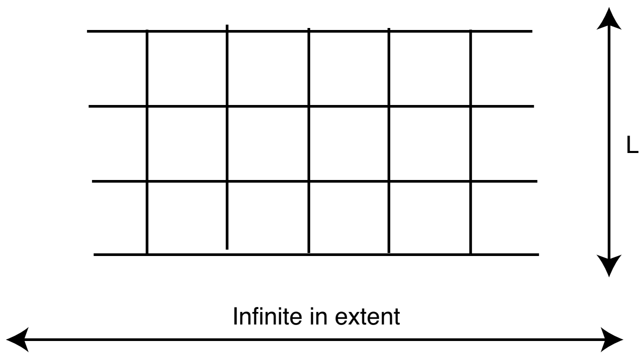
Important

All $T \neq 0$ transitions are “classical” by definition—even in a highly quantum system like superfluid Helium.

Close to $T \neq 0$ critical point quantum fluctuations are important on a microscopic scale, while classical thermal fluctuations dominate on macroscopic scales—can be described by classical statistical mechanics with an effective Hamiltonian of an order parameter field.

How does one determine QCP from finite temperature measurement?

Ising model on a strip ($\infty \times L$): a digression



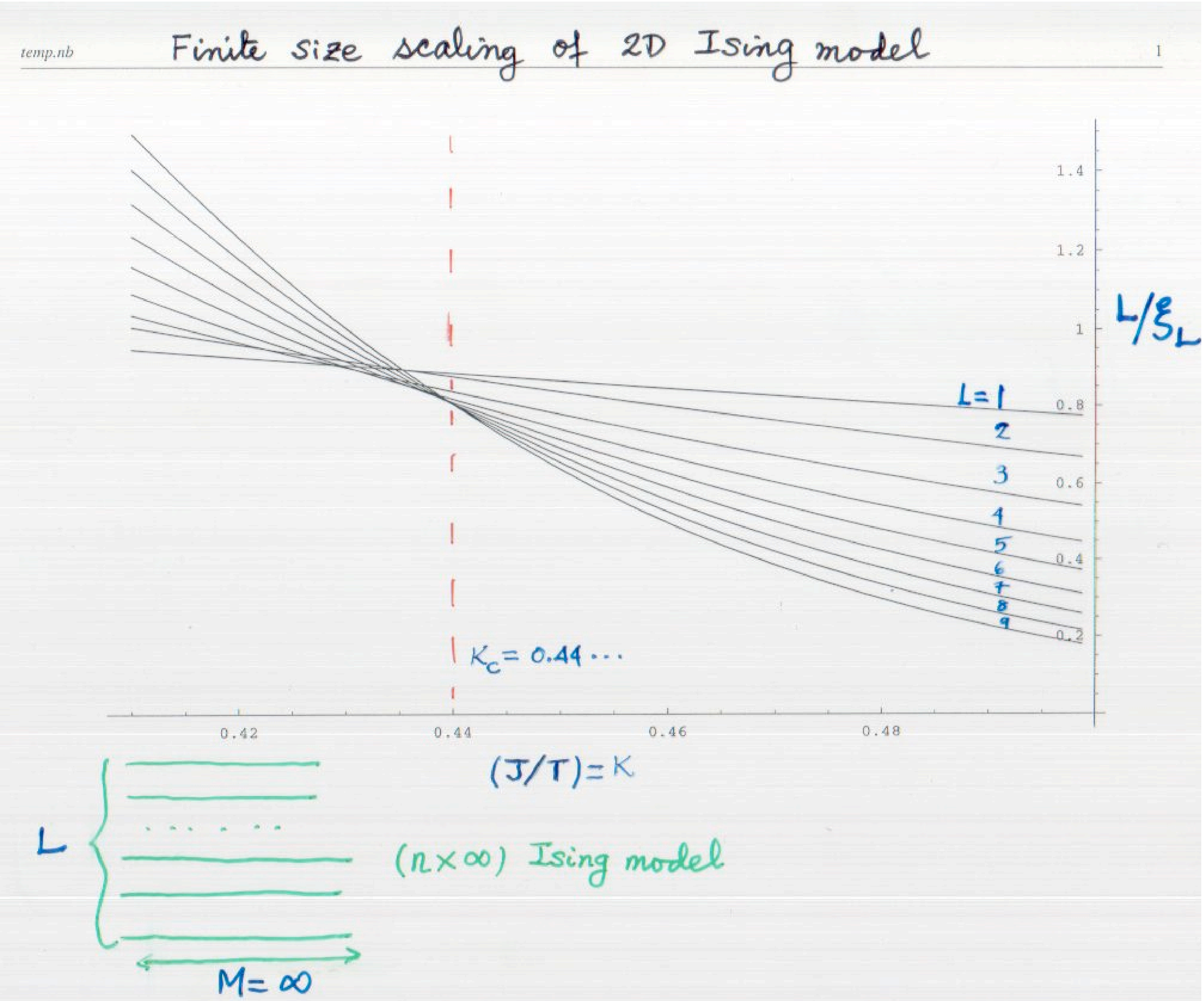
$$H = -J \sum_{\langle ij \rangle} S_i S_j, \; S_i = \pm 1$$

- No phase transition as long as L is finite.

- For $L = \infty$, the phase transition is at $K_c = J/k_B T_c = 0.44068\ldots$ and $\xi \sim |T - T_c|^{-\nu}$, with $\nu = 1$.

- How can we find this transition from strips of finite width L?

Explicit finite size scaling in Ising model in a strip

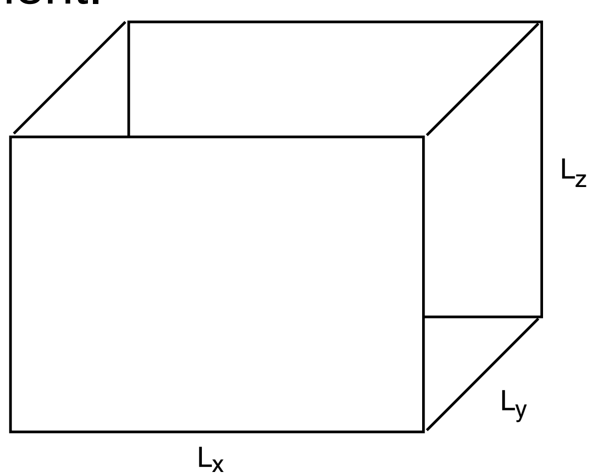


Finite-size scaling hypothesis states that

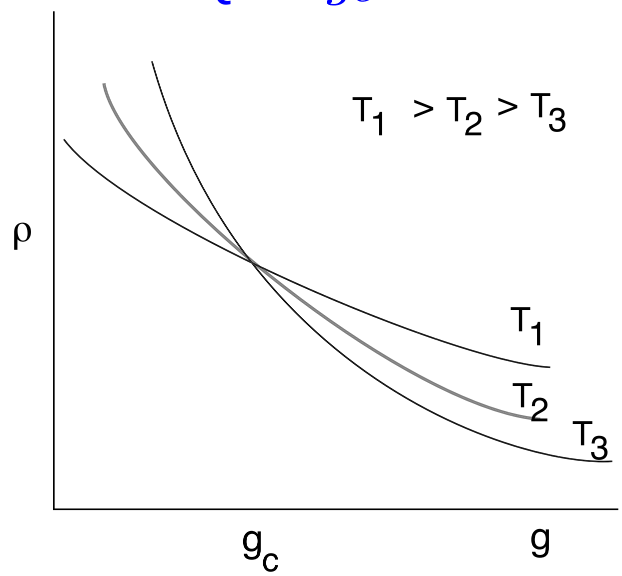
$$\frac{L}{\xi} = A + BtL^{1/\nu} + O(t^2),\ t = \left|\frac{T - T_c}{T_c}\right| \to 0.$$

If we tune T to T_c of the infinite system, (L/ξ) will be independent of the width of the strip L. Converesely, if the curves for (L/ξ) have a unique crossing point, it must be the T_c of the infinite system.

A similar finite-size scaling trick works for finding QCP. One can show that a $2D$ quantum system can be viewed as a finite slab, where $L_x = L_y = \infty$, but $L_z = 1/T^{1/z} \to \infty$ as $T \to 0$, where z is the dynamical exponent.

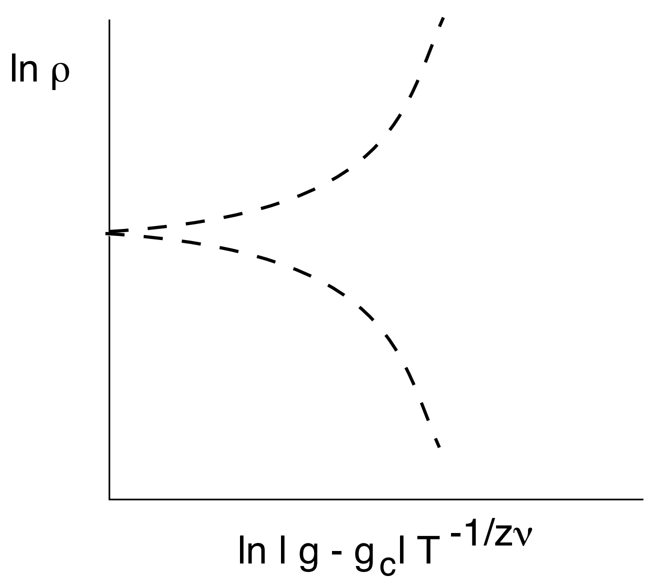


Determination of the QCP g_c



Data collapse

Universal scaling functions



Stanford group: Kapitulnik et al.

Experiments:

amorphous superconductor: $Mo_{43}Ge_{57}$

T_c of bulk ~ 7 K

T_c of 30 Å film ~ 0.5 K

H_{c2} ~ 1.4T

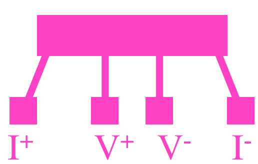
□(0) ~ 100 Å

□(0) ~ 8000 Å

$\ell \sim$ 2-3a_0 ~3-6Å

homogeneous to < 20Å

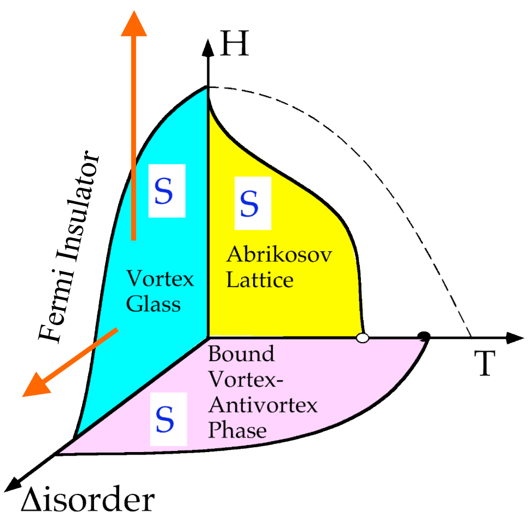
Experiments done on patterned films in a dilution refrigerator



Courtesey: A. Kapitulnik.

Two possible Superconductor-Insulator Transition at T=0:

Superconductor --> Fermi-Insulator

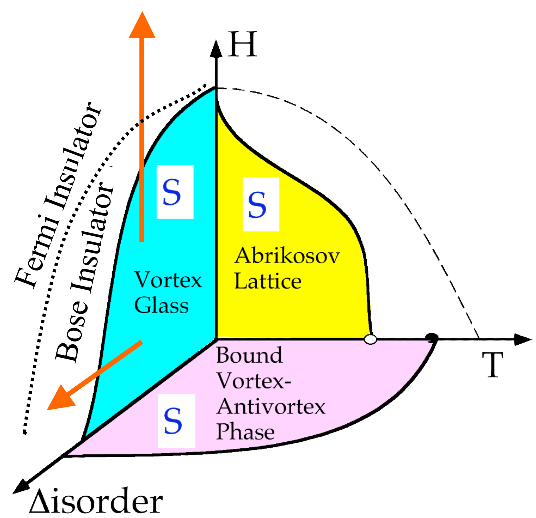


1) Superconductivity is destroyed by disappearance of Cooper pairs altogether. Cooper attraction is reduced due to Large Coulomb interaction. *

*A.M. Finkelstein, JETP Lett. 45, 46 (1987).

This model however neglects quantum fluctuations of the Bosonic field! **Free electrons exist**!

Superconductor --> Bose-Insulator



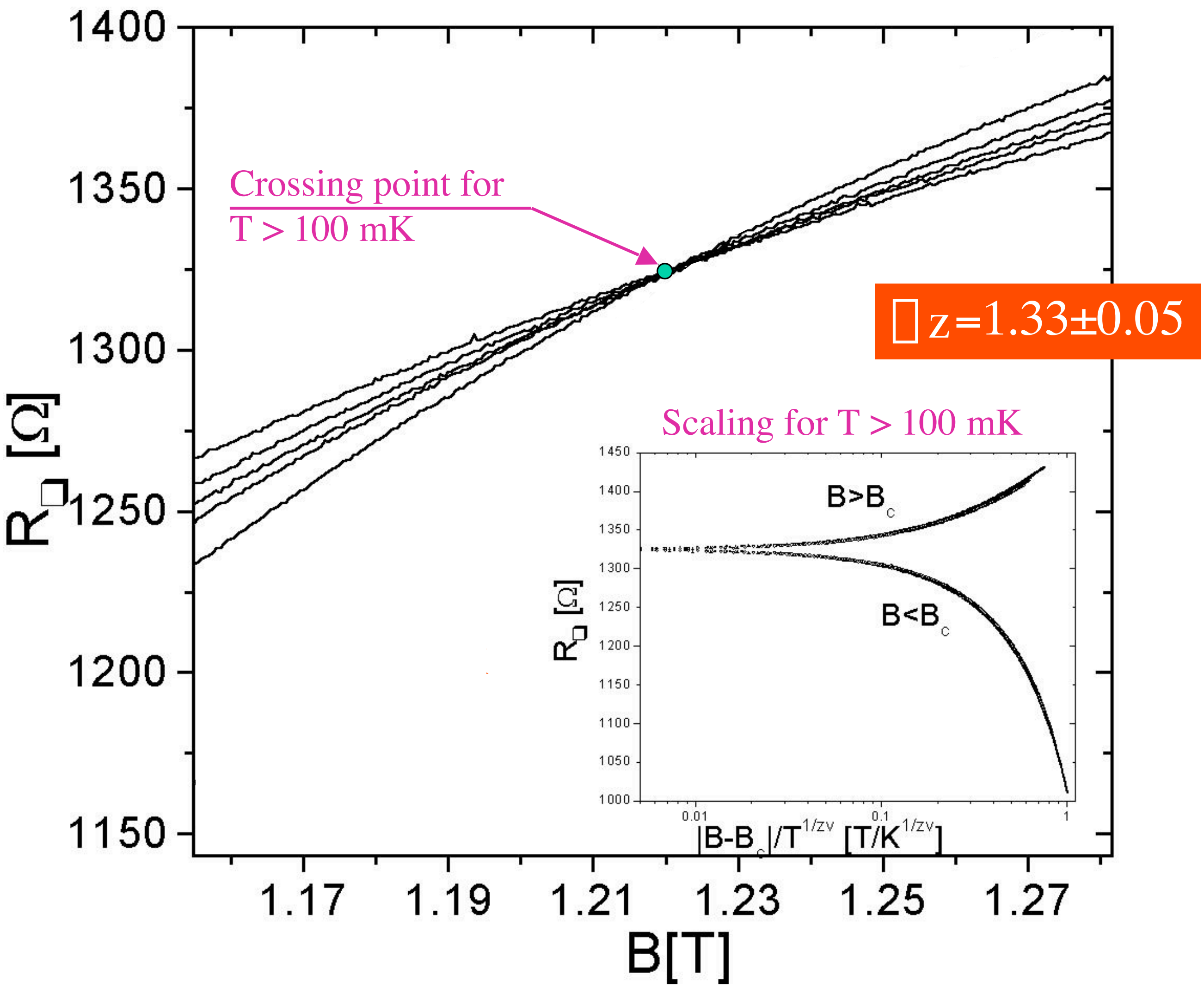
2) Bosons in a random potential. Pairs can become localized due to coulomb repulsion.
Equivalent to array of Josephson-Junctions (E_J vs. E_C). *

* M.P.A. Fisher, Phys. Rev. Lett. (1990).

No free electrons exist!

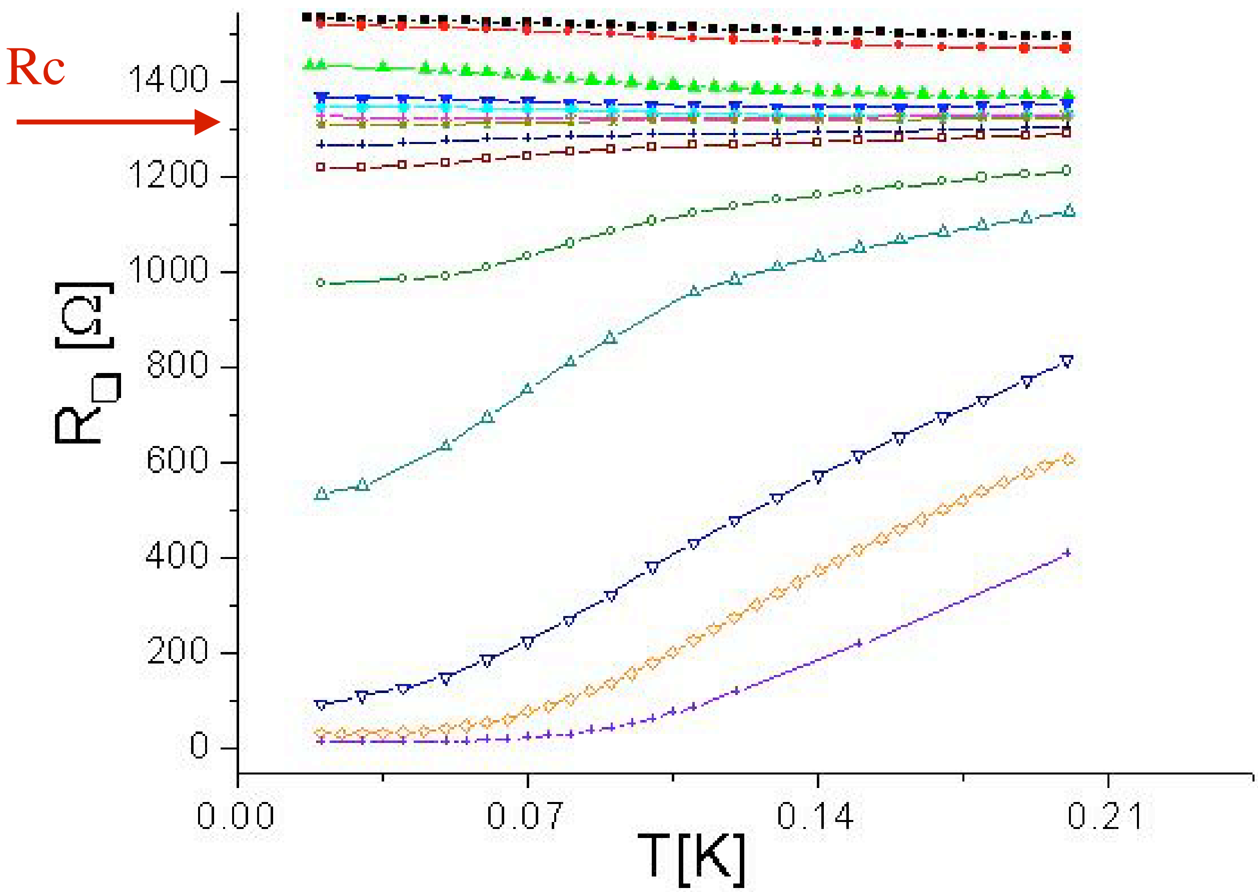
Yazdani and Kapitulnik: apparent superconductor-insulator transition.

Crossing point and scaling



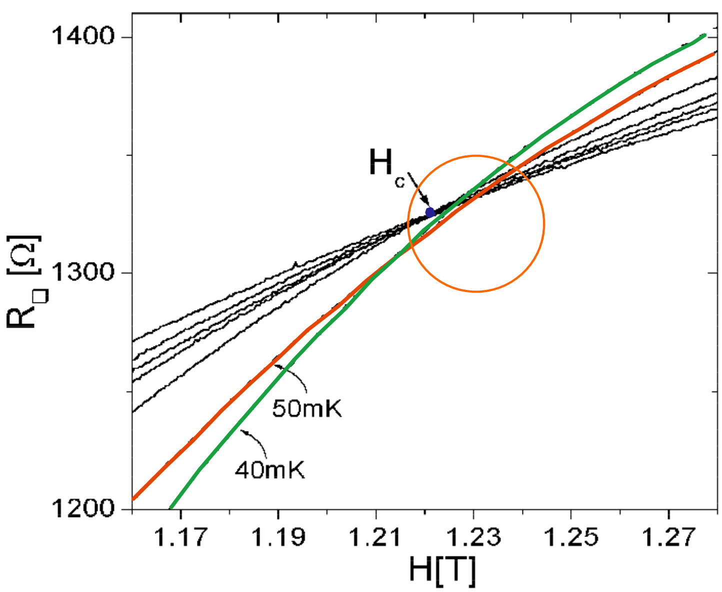
Mason and Kapitulnik: apparent dissipative metallic state at low temperatures.

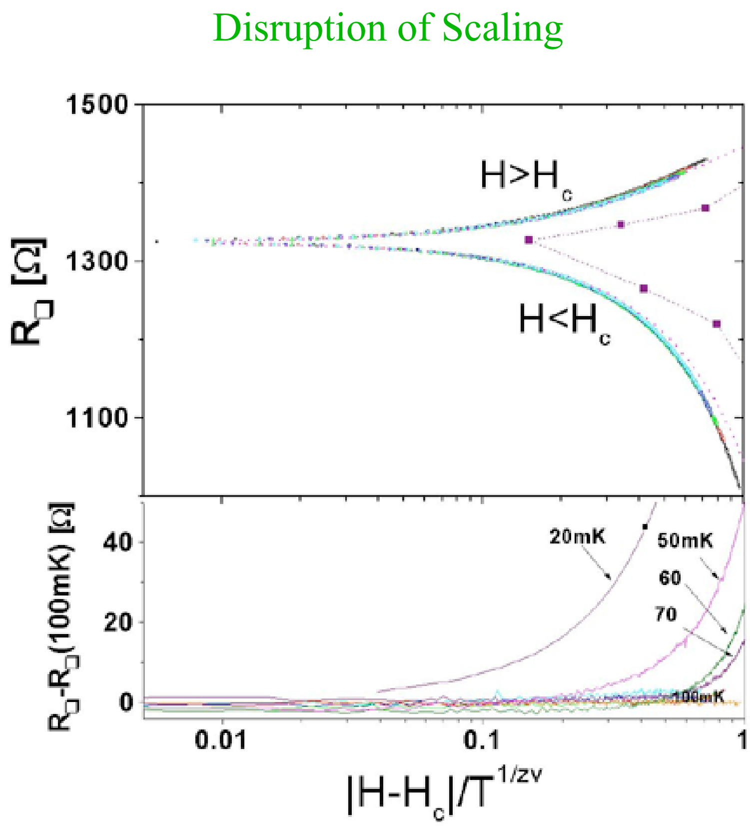
Resistive transitions for T > 20 mK



Mason and Kapitulnik: breakdown of scaling.

Broadening of Crossing at Low T:

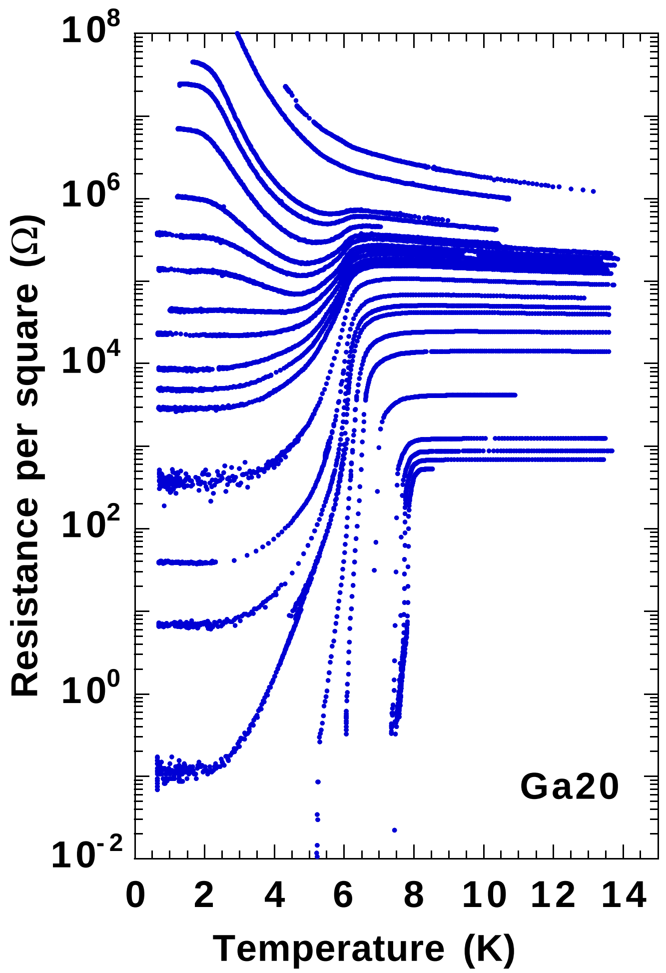




Courtesey: A. Goldman

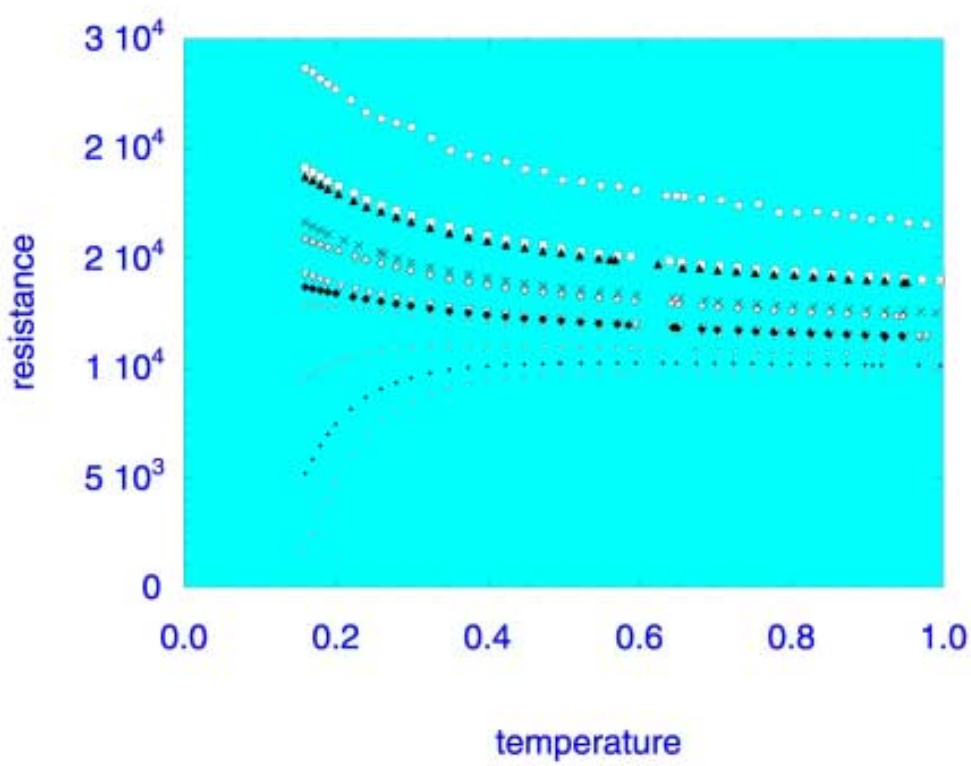
Jaeger ~ 1989

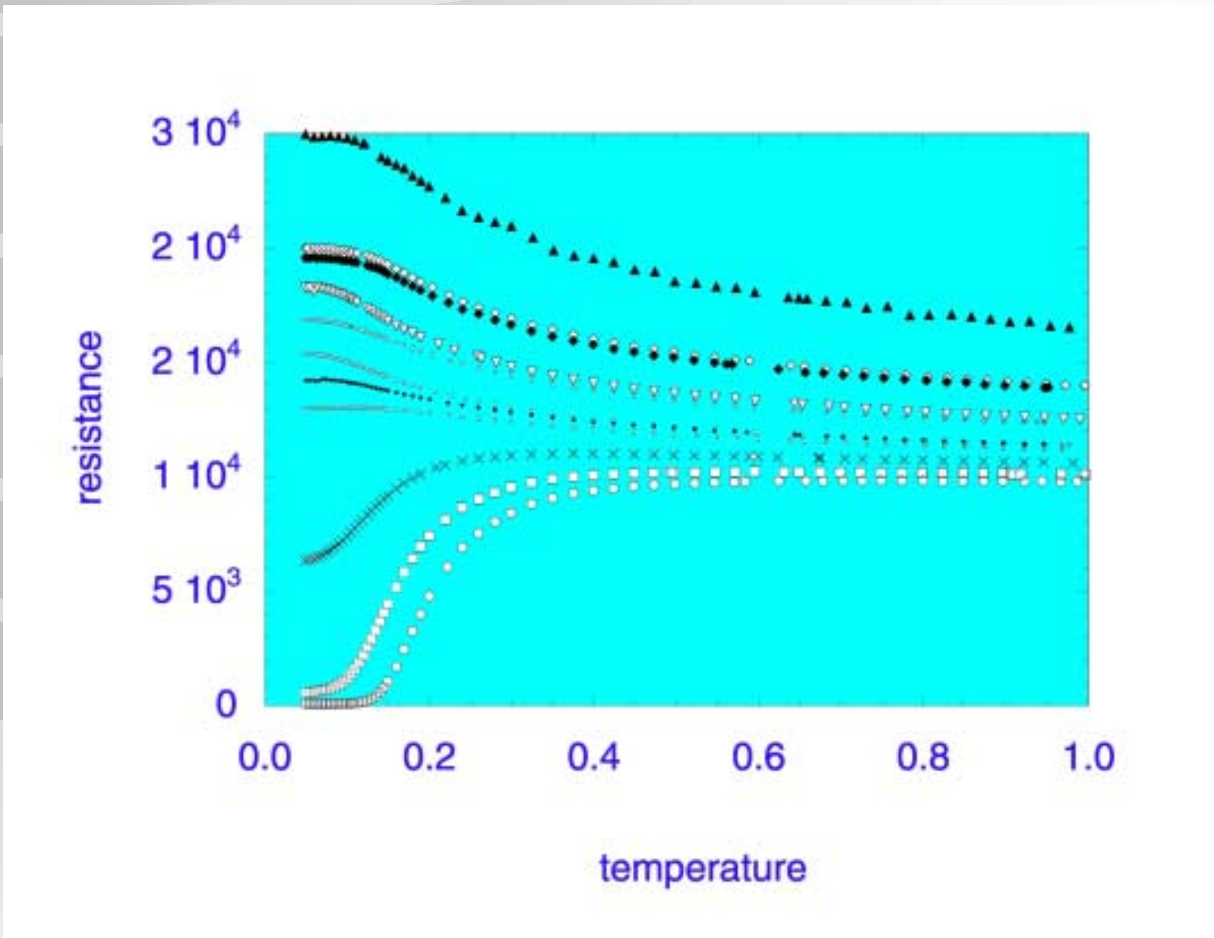
- **Amorphous Ga grown on a glass substrate.**
- **Clear evidence of metallic behavior at low temperatures.**
- **Also have local superconductivity.**
- **Motivated early work of Chakravary*et al*. On dissipation-controlled transitions.**



Goldman et al. : disorder tuned superconductor-insulator transition in a-Bi on a-Ge film.

Resistance vs. Temperature at Various Thicknesses





All films appear metallic in the limit of zero temperature! (actually measured down to the bottom T << 50mK)

Dissipation and its Importance

- Ubiquity of a low temperature metallic phase and hence disssipation at low temperatures
  - Two-dimensional metal-insulator transition in Si-MOSFET
  - Field tuned superconductor-insulator transition, with an intermediate metallic phase
  - Quantum Hall-insulator transition
  - Superconductor-metal-insulator transition (B=0)
  - Sinking of the extended states
  - Low temperature saturation of phase breaking times in transport
  - Possibly others

A d-dimensional quantum system coupled to a dissipative heat bath is likely to change the universality class of the QCP. The key reason is that the Caldeira-Leggett Ohmic heat bath induces a special non-local temporal interaction, while most renormalization group analyses are based on local interactions.

- Ising model in a transverse field coupled to dissipation: Philip Werner (ETH), Matthias Troyer (ETH), and S. C. Extensive numerical simulations indicate that the universality class is dramatically altered.

- Quantum criticality is a both precise and fascinating concept. Rigorous results are theoretically known, and yet experimental confirmations in a clean system are hard to come by.

- There seems to be a ubiquitous metallic state at low temperatures. Its origin is unknown, and its proper theoretical treatment is also unknown.